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Stability and Size of Galaxies from Planck's Constant

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Stability and characterisitic geometrical and kinematical sizes of galaxies are strictly related to a minimal characteristic action whose value is of order h, the Planck constant. We infer that quantum mechanics, in some sense, determines the structure and the size of galaxies.

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From a classical point of view, there are no arguments capable of completely explaining the stability and the size of galaxies: the only assumption is that they are considered to be relaxed (and virialized) systems where gravity is the overall interaction [1]. Such a force is considered as a "Newtonian" interaction and the confining potentials, due to the mutual attractions of stars (and the other components as dust and gas clouds) can have several forms. For example, logarithmic potentials well describe the regular motion of stars before the onset of chaos [2]. In any case, the "stability is an assumption" and sizes are deduced from observations. Actually, the problem is extremely involved since galaxies undergo environmental effects, being never isolated systems; they always belong to large gravitationally bound systems as loose and tight groups, associations or clusters of galaxies and the observational times are so short that the overall dynamics can be only inferred [1],[3]. Besides, galaxies have to be related to some cosmological model and, due to cosmological evolution of large scale structures, they should be connected to some theory of primordial perturbations [4],[5]. For these reasons, it is not senseless to ask for some quantum signature in the today observed galaxies [6]. The main point, however, is to connect the estremely large size of galaxies ($\sim$ 10kpc) with the extremely small numbers of quantum mechanics ($h \sim 10^{-27}$ erg sec).

In this letter, we want to show that, for a given galaxy, a minimal characteristic action is of the order of h and, furthermore, the onset of chaos [7] is prevented if and only if the characteristic sizes of a galaxy are related to the Planck constant. In other words, it is the quantum signature which stabilizes the galaxies; furthermore it gives rise to their characteristic sizes, where, by "sizes", we intend geometrical and kinematical quantities which assignes a galaxy.

This result is not particular since the collective features (in particular the stability and the confinement properties) of several mesoscopic and macroscopic systems can be explained only by invoking quantum

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coherence on large scales (see for example [8],[9]). Very famous examples of this new trend in physics are the high T_c superconductivity systems or the optical fibres. Besides, it is possible to show that h is the characteristic action for several macroscopic systems. The scheme is: given a classical law of force $F(R)$, describing a system where N particles are interacting on a length scale R, a characteristic action of order h is recovered.

The "classical" force $F(R)$ can be the electromagnetic interaction of accelerator beams, the strong interaction of quark aggregates [10], or the Newtonian interaction acting on all the nucleons which are present in the Universe [11]. Taking into account generalized theories of gravity [12],[13],[14] which give corrective terms to the Newtonian potential in the weak energy limit, it is possible to show that any gravitationally bound system, where gravity is the only overall interaction, undergoes this scheme [6].

A heuristic argument can be given considering the total action for a bound, virialized system where N is the number of constituents. Let E be the total energy so that the system is bound and stable. Let $\mathcal{T}$ be the characteristic time of the system (*e.g.* the time in which a particle crosses the system, or the time in which the system evolves and becomes relaxed). Combining these two quantities, we get

$$\mathcal{A} \cong E\mathcal{T}\,, \tag{1}$$

which is the total action. The only hypothesis which we need is that the system could undergo a time–statistical fluctuation, so that the characteristic time τ for the stochastic motion per particle will be [10],[11]

$$\tau \cong \frac{\mathcal{T}}{\sqrt{N}}\,. \tag{2}$$

This hypothesis naturally emerges from the fact that a galaxy can be treated as a statistical system [1].

Immediately, we can define an energy per particle

$$\epsilon \cong \frac{E}{N}\,, \tag{3}$$

and then a characteristic unit of action per particle is

$$\alpha = \epsilon\tau \cong \frac{\mathcal{A}}{\sqrt{N^3}}\,. \tag{4}$$

These formal considerations can be applied to bound physical systems where the degrees of freedom have acquired the same energy (that is are virialized). It can be shown that for several systems (among them also the whole observable Universe), it is

$$\alpha \simeq h\,, \tag{5}$$

with an error of approximatively an order of magnitude [10],[11].

Let us now consider galaxies. The onset of chaos [15], in a realistic galactic potential, is for an energy per unit of mass of the order 10^{15} (cm/sec)^2 while the period of a galactic rotation, which can be assumed as a characteristic time, is about

$$\mathcal{T}_{rot} = 3\times 10^{15}\text{sec}\,. \tag{6}$$

The total mass of a typical galaxy is

$$M \cong 2\times 10^{44}\text{gr}\,. \tag{7}$$

From Eq.(1), combining these numbers, we get

$$\mathcal{A} \cong 10^{74}\text{erg sec}\,. \tag{8}$$

The number of nucleons in a star of a solar mass is $\sim 10^{57}$ and then, for a galaxy,

$$N \cong 10^{68}\,. \tag{9}$$

Introducing these numbers inside Eq.(4), we get, with an error of an order of magnitude, that *the characteristic unit of action for a galaxy is of the order of Planck constant.* It is interesting to stress the fact that also if dark matter is considered into dynamics the result does not change dramatically since the mass to luminosity ratio is of the order $10 \div 100$.

It is interesting to note that the values which we have used are on the boundary for the onset of chaos and the galaxy is assumed stable. In other words, the stability of the system is related to the quantum mechanics. Furthermore, the stability and the connection to quantum mechanics scale with the number of particles.

More formally, the characteristic unit of action can be derived for a system where a classical law of force $F(R)$ acts on the constituents of mass m over a global size R. If the system is stable and virialized, the characteristic work done by the system is

$$\mathcal{L} \cong mv^2\,, \tag{10}$$

and then

$$\mathcal{L} \cong NF(R)R\,. \tag{11}$$

Using Eqs.(2), (3) and (4), one gets

$$\alpha \cong m^{1/2}R^{3/2}\sqrt{F(R)}\,, \tag{12}$$

indipendently of the type of force. In all cases one obtains $\alpha \cong h$ [8],[10]. In particular, the result holds for gravitationally bound systems which can be globular clusters, galaxies, and clusters of galaxies [6], up to the whole universe [11]. In these cosmological cases, the gravitational coupling, *i.e.* the Newton coupling G_N must scale with the distance as several modified quantum theories of gravity imply [12],[13],[16],[17]. However, the modification of G_N is small and Newtonian gravity holds in the weak energy limit. Confirmations of this scheme are coming from satellites' measurements of long range acceleration [18]. Gravitational potentials like

$$V(R) = -\frac{G(R)M}{R}\,, \tag{13}$$

with

$$G(R) = \chi G_N \left(\frac{R}{R_0}\right)^{\eta} \ln\left(\frac{R}{R_0}\right)\,, \tag{14}$$

or

$$G(R) = G_N\left[1 + a_0 \exp(-R/R_0]\right)\,, \tag{15}$$

well describe this situation. The parameters χ, η, a_0 depend on the modified theory of gravity used [14],[19]. R_0 can be assumed, for galaxies, of the order $\simeq 10$kpc [17].

An important point must be stressed. All these simple hypotheses do not work for a single star which, from our point of view, is not properly a gravitationally bound system. In fact, nuclear and electromagnetic interactions contribute to the stability of the system so that it cannot be simply schematized only with a classical force acting on it.

This result, as we said above, holds also for other mesoscopic and macroscopic systems as accelerator beams, quark condensates and Bose–Einstein condensates [10]. The rule, thus, seems general and it works also at astrophysical scales as those of galaxies. In all cases these situations, involving complex

aggregates which exhibit a nontrivial interplay between mechanical or quantum mechanical effects and thermodynamical and statistical effects, one can suitably define effective scales of length, velocity, and energy, as well as effective temperatures. We will now show how all these allow to derive some interesting results on the geometrical size of galaxies, on the average thermal velocities and wavelenghts for the galaxies.

Let us first introduce the “emittance”, which is a scale of length (or, equivalently, of “temperature”) related to a given complex, correlated system (such as a Bose condensate or a charged particle beam in an accelerator [10]). It can be defined as

$$\mathcal{E} \cong \lambda_c \sqrt{N}\,, \tag{16}$$

where

$$\lambda_c = \frac{h}{mc}\,, \tag{17}$$

is the Compton length associated to the constituent particle m.

In the case of galaxies, $m = m_p \cong 10^{-24}$gr, which is the proton mass. N is given by Eq.(9) and we get

$$\mathcal{E} \simeq 10^{22}\text{cm} \simeq 10\text{kpc}\,, \tag{18}$$

which is a typical scale of length for a normal galaxy. This fact means that the quantum parameter λ_c and the number of constituents N determine the astrophysical size $\mathcal{E}$ which is related to the stability of the system. It is interesting to stress that this is the typical size where the rotation curve of a galaxy can be assumed flat [1] and, in some sense, where the halo and the disk stabilize each other. If the characteristic time is given by Eq.(6) and the geometrical scale is (18), we get

$$v \simeq 10^{7}\text{cm/sec}\,, \tag{19}$$

which is a typical rotational velocity for the outer components of a galaxy. A further interesting quantity is the time after which the system can be considered virialized. It is

$$\mathcal{T}_{vir} \simeq 10 \div 100\mathcal{T}_{rot} \simeq 1 \div 10\text{Gyr} \simeq 10^{16\div 17}\text{sec}\,. \tag{20}$$

Considering also the typical maximal extension of the halo which can be assumed

$$R \simeq 1 \div 10\mathcal{E} \cong 10 \div 100\text{kpc}\,, \tag{21}$$

we get

$$v \simeq 10^{5\div 7}\text{cm/sec}\,, \tag{22}$$

which is the range where are placed all the typical velocities of a normal galaxies, i.e. from the dispersion of velocities of stars ($\sim 10^5$cm/sec) to the circular speed of a star in the disk ($\sim 10^7$cm/sec). We stress again that all these quantities are, in some sense, related to the Planck constant h.

From a thermodynamical point of view, the fluctuative time (2) can be defined as the ratio between the typical quantum mechanical size h anf the typical Boltzmann “size” $k_B T$, *i.e.*

$$\tau \cong \frac{h}{k_B T}\,, \tag{23}$$

then the temperature of the system is

$$T \cong \left(\frac{h}{k_B}\right)\frac{\sqrt{N}}{\mathcal{T}}\,. \tag{24}$$

Let us note that Eq.(24) can be rewritten in the form

$$(k_B T)\mathcal{T} \cong h\sqrt{N}\,, \tag{25}$$

which defines the thermal unit of emittance and also determines, dividing by mc, the typical length range of interaction, Eq.(16).

Let us now consider a galaxy as a thermodynamical system, then

$$\frac{1}{2}m_p\langle v^2\rangle = \frac{3}{2}k_B T\,. \tag{26}$$

Using Eq.(24), we get

$$\langle v^2\rangle \simeq \left(\frac{3h}{m_p}\right)\frac{\sqrt{N}}{\mathcal{T}_{vir}}\,, \tag{27}$$

and then the result (22) is recovered for velocities. Besides this "thermal velocity", it is straightforward to define a "thermal wavelength". Using the above results, we can write, in general,

$$\lambda_T \cong \frac{R}{\sqrt{N}}\,, \tag{28}$$

and

$$\langle v\rangle \cong \frac{R}{\mathcal{T}} \cong \frac{\lambda_T}{\tau}\,, \tag{29}$$

considering Eqs.(23) and (24), we get

$$\lambda_T \cong \frac{h}{\sqrt{m_p k_B T}}\,, \tag{30}$$

which is of the order of atomic size as it must be for a proton.

In conclusion, we can say that the geometrical size, the kinematic and the stability of galaxies are strictly related to quantum mechanics. In other words, it seems that the structure of galaxies is ruled by quantum mechanics which prevents the onset of chaotic behaviour and, in some sense, the dissipation of the constituents. Furthermore, it seems that the number of constituents (the nucleons inside the stars) the geometrical global size R, the classical law of force $F(R)$ have to combine in order to give a characteristic action of order h to stabilize the systems. A further step that the authors are going to face is to understand the dynamics of such a feature and its connection to the cosmological evolution.

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